JUNE

NCTM NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

For how many three-digit numbers between 100 and 999 do the first and last digits differ by 3?

1

Exactly three of the numbers in the set

$$\{12, 14, 15, 20\}$$

divide evenly into some number n. Which number will not divide evenly into n?

2

Calculate the average of the four prime numbers closest to 58.

3

What is the largest number that will divide every number greater than 1000 that ends in 768?

4

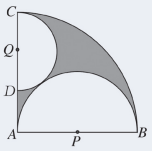
If

$$2a + 5a + 8a + 11a = \frac{b}{2} + \frac{b}{5} + \frac{b}{8} + \frac{b}{11},$$

find the ratio of a to b.

5

Arc BC is a quarter circle with radius 2 and center A. P and Q are the centers of mutually tangent semicircles with diameters AB and CD, respectively. Compute the shaded area.



6

Which of the following is larger?

$$\sqrt[99]{99!}$$

or

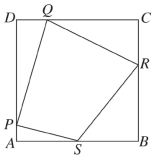
$$\sqrt[100]{100!}$$

7

For how many four-digit numbers does the sum of the digits equal 9?

8

Given square $ABCD$ with side length 8, find the area of $PQRS$ if $AP = 1$, $DQ = 2$, $CR = 3$, and $BS = 4$.



9

Solve the following:

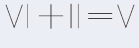
$$3^{2x+1} - 3^{x+3} - 3^x + 3^2 = 0$$

10

The numeral 65 in base x represents the same number as 56 in base y. Find the smallest value of $x + y$.

11

Eleven toothpicks can be arranged to form the following incorrect statement involving roman numerals. Move just one toothpick to make the equation correct. (There are several solutions.)



12

In an orchestra, 23 people can play stringed instruments, 18 can play brass, and 12 can play percussion. Further, 10 of the performers can play both strings and brass, whereas 6 can play both strings and percussion. If no one can play all three types of instruments, what are the maximum and minimum numbers of people in the orchestra?

13

A salesman took a $20 taxi ride to get to his first client, but there he tripled the amount of money he had left. His ride to the second client cost $12, and there he only doubled what he had left. The cost of the ride to his third client was $17, and again he doubled the amount of money he had. If the salesman ended with $626, how much money did he have initially?

14

A square and an equilateral triangle have the same perimeter. Compute the ratio of the area of the triangle to the area of the square.

15

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16

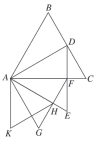
Square $ABCD$ has two quarter circles drawn in its interior. The centers of the circles are two consecutive vertices of the square, and the radius of each circle is the length of a side of the square. If the side of the square is 8, what is the distance from the intersection of the quarter circles to the nearest side of the square?

17

Find the sum of all three-digit integers whose units digit is 7.

18

$\triangle ABC$ is equilateral with altitude AD. Use the altitude to form a second equilateral triangle, $\triangle ADE$. Use the altitude of $\triangle ADE$ to form a third equilateral triangle, $\triangle AFG$, and then use the altitude of $\triangle AFG$ to form a fourth equilateral triangle, $\triangle AHK$. Find the ratio of the area of $\triangle ABC$ to the area of $\triangle AHK$.



19

How many different types of cross sections can be made by cutting a cube with a plane? Two examples are a square and an equilateral triangle.

20

Find the minimum value of the following expression:

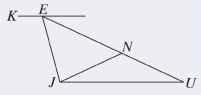
$$|x-3| + |x-1| + |x| + |x+2| + |x+4|$$

21

Three circles—P, Q, and R—are externally tangent to one another. The radii of the circles are 5, 12, and 13, respectively. Name the smallest angle in $\triangle PQR$.

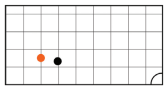
22

In $\triangle EJU$, $m\angle NJU = 25°$, and $m\angle JNU = 130°$. If $\overline{KE} \parallel \overline{JU}$ and $NU = EJ$, find $m\angle KEJ$.



23

A coordinate grid is placed on a 4.5 × 9-ft. pool table. The object is to sink the ball located at (2, 1.5) in the hole located at (9, 0). Since a direct shot is blocked, the shooter must bank her shot off an edge opposite the hole. Find the coordinates of the point she must hit to make the shot.



24

Find two numbers a and b such that

$$(a - b) : (a + b) : ab = 1 : 5 : 24.$$

25

The line $y = mx + 3$ is tangent to the ellipse whose equation is

$$4x^2 + y^2 = 4.$$

Find the value of m^2.

26

Find the ordered pair of integers (x, y) that satisfy the equation

$$3^{2x} - 2^{2y} = 17.$$

27

The coordinates of three points are A (–3, 2), B (25, 10), and C (k, k). Find the value of k such that $AC + BC$ is a minimum.

28

The coordinates of three points are A (–3, 2), B (5, 15), and C (k, k). Find the value of k such that $AC + BC$ is a minimum.

29

Solve the following equation for x:

$$\sqrt{x + \sqrt{x + 271}} + \sqrt{x - \sqrt{x + 271}} = 6$$

30